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THE EFFECT OF FLEXIBLE MOUNTINGS UPON THE RESONANT SPEEDS OF MACHINES HAVING UNBALANCED MOTORS

Hugh Trent Mackay

The Effect of Flexible Mountings upon the Resonant Speeds
of Machines Having Unbalanced Rotors

By

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THESIS

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INTRODUCTION

The subject of this paper was suggested by Assistant Professor C. F. Garland of the Mechanical Engineering Department of the University of California. It is an outgrowth of a graduate course in the dynamics of machinery presented at the University.

Any machine having a rotor is usually considered to be, in some degree, unbalanced. Therefore there will be a speed (or speeds) at which resonance will occur due to the frequency of the periodic force of the unbalanced rotor coinciding with the natural frequencies of the system concerned. Such speeds are termed resonant or critical speeds. The mountings of most machines are flexible in some measure. Even if the immediate foundation be considered absolutely rigid either the building or the ground upon which it rests may possess flexibility. In most cases this flexibility is not sufficient to appreciably affect the critical speed of the machine. There are cases, however, in which the mountings are relatively quite flexible for instance a diesel engine placed in a submarine.

It is possible that the flexible mounting would be a source of trouble. But it is also both conceivable and possible that the resulting phenomenon be put to advantageous use. Thus in one case a turbine was found to be operating very near its critical speed. The critical speed was then changed to a value removed from the proximity of the operating speed by making the supporting structure more elastic.

Stodola (reference 3) mentions the possibility of the resonance of a rotor and its foundation. He concludes that the "sympathetic vibrations" will not lead to violent vibrations. Hort (reference 5) considers the problem in relation to the transfer of the machine vibrations to the foundations. He shows that at resonant speeds a large amount of energy is absorbed by the vibrating foundation. Rodgers (reference 6) gives equations for the amplitude of vibration of such a system in a general discussion of factors affecting critical speeds. Timoshenko (reference 2) considers the critical speeds of a machine somewhat similar to that taken up in this article.

In conducting the investigation for this paper amplitudes of vibration of an unbalanced rotor on a flexible mounting were measured at various speeds. Then the amplitudes of vibration were again measured over the same range of speeds with the mounting made rigid. Curves of the amplitude versus speed were plotted. This procedure was repeated for various values of the mass of the rotor and of the elastic constant of the mounting. At certain well defined speeds the amplitudes were found to be quite large. These particular speeds are the "resonant speeds" on which this study is focused.

Theoretical equations were then derived for the amplitude of vibration of the rotor by considering the apparatus as a coupled two body system with two degrees of freedom excited by the centrifugal force of the unbalanced rotor. The effect of friction was neglected. The expression for the resonant speeds was obtained from the equation for the amplitude of vibration. A dimensionless

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quantity was obtained by dividing these resonant speeds by the expression for the critical speed of the rotor when the mounting is rigid. Curves were plotted from the expression thus obtained using as a variable the ratio of the mass of the rotor to the mass of the part of the supporting structure that vibrates and using as a parameter the ratio of the elastic constant of the mounting to the elastic constant of the rotor. The derivations of all expressions used are presented under the heading "Derivation Of Equations".

The experimental and theoretical results are shown on curve sheets 1 and 2. The problem of this paper is considered in the various references as before mentioned. But in no reference that was consulted is there any consideration of a comparison of the experimental and theoretical results.

The flexibility of the mounting of the apparatus used for experimentation could be varied in the vertical plane only. Therefore this paper is limited to consideration of vibrations in the vertical plane.

CONCLUSIONS

(1) An unbalanced rotor on an elastic shaft carried by an elastic mounting has two resonant (critical) speeds. Both of these speeds are different from the one critical speed of the same rotor on a rigid mounting.

(2) Of the two critical speeds of a rotor with an elastic mounting one is above and one below the critical speed of the same rotor with a rigid mounting.

(3) The two critical speeds of a rotor with elastic mounting correspond to the two natural frequencies of an elastically coupled two mass system.

(4) For each of the two critical speeds the ratio of critical speed with flexible mounting to the critical speed with rigid mounting increases with increase of the ratio of the mass of the rotor to the mass of the base. (See curve sheets 1 and 2.)

(5) For each of the critical speeds the ratio of critical speed with flexible mounting to the critical speed with rigid mounting increases with increase in the ratio of the elastic constant of the flexible mounting to the elastic constant of the rotor. (See curve sheets 1 and 2.)

(6) The effect of damping on the numerical value of the critical speeds was very slight in the equipment used for experimentation.

DESCRIPTION OF APPARATUS

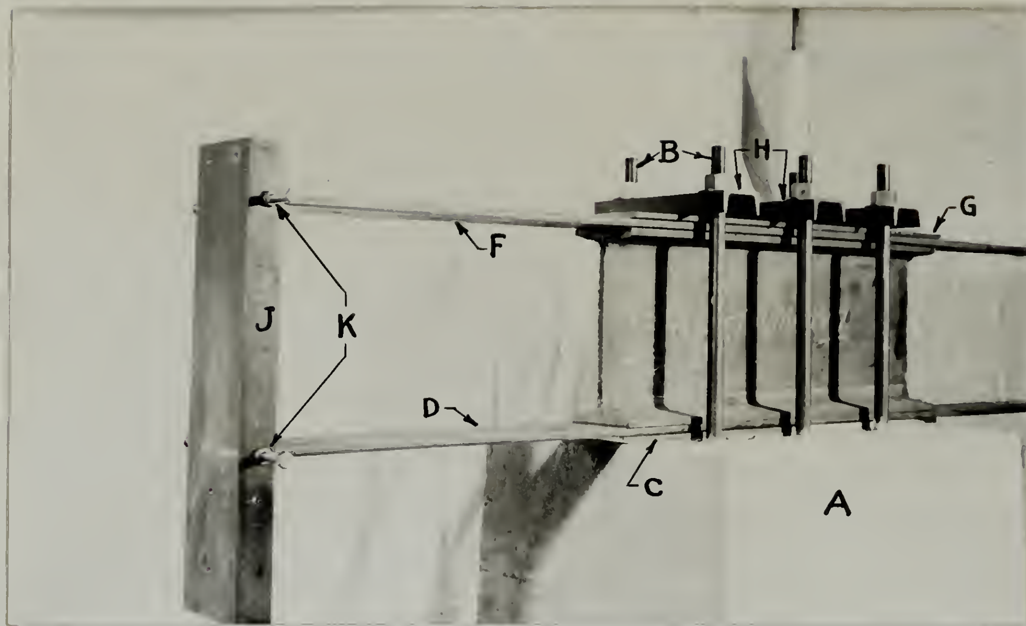
The following description refers to figures 1 and 2.

In assemblying the apparatus a concrete foundation A, 26 inches high, was cast. Imbedded in the foundation and extending 18 inches above it are six 1-inch bolts B. Between the rows of bolts a 3/8-inch steel plate was placed and leveled. Two ½-inch steel bars D and F, 5 inches wide and 72 inches long, and spaced by a 30-inch length of 12-inch I beam were clamped to the foundation by means of the channel sections H, the plate G and the bolts B. Thus the bars D and F act as cantilever springs and supply flexibility to the mounting. The length of the springs can be varied by sliding them relative to the foundation after releasing the pressure on them.

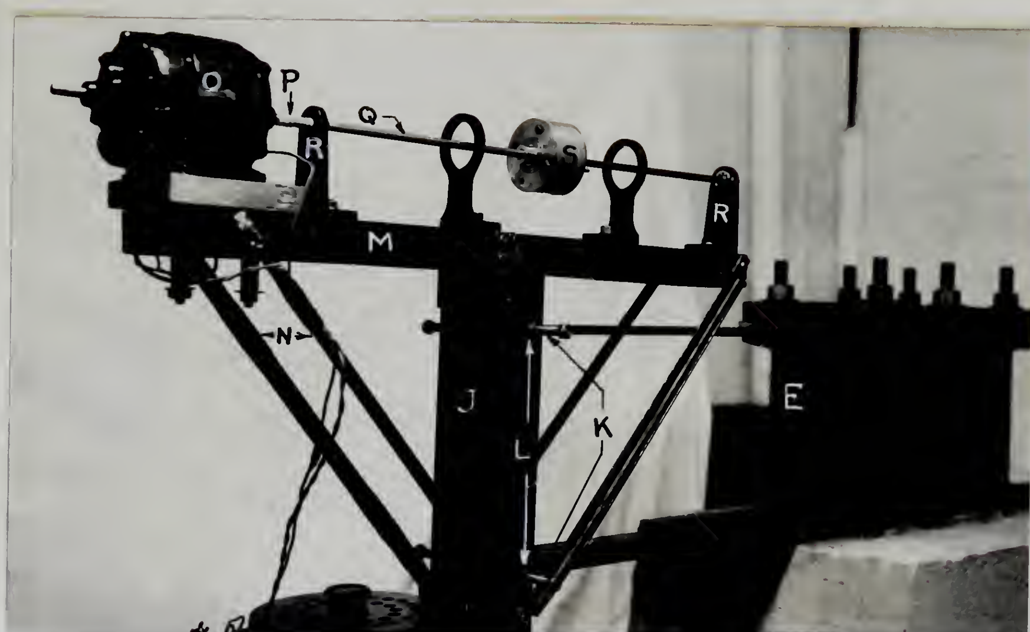
A vertical member J is attached to the free ends of the springs D and F by means of conical pivots K. The base M is secured to the top of the vertical piece J and is braced by bars of ¼-inch angle iron N. The base is made of two sections of 2-inch angle iron joined together with webs of 1/8-inch plate welded in place. A ½-inch slot extending the length of the base provides an anchorage for the bolts which hold the motor and bearing pedestals in place.

The ¼-horsepower motor O is D.C. shunt wound and has a rated speed of 1750 r.p.m. The shaft of the motor is connected to the shaft carrying the rotor by a short section of ½-inch rubber tubing. This provides a satisfactory flexible coupling.

The shaft Q is of cold rolled steel, 7/16 of an inch in



Elastic Mounting
Figure 1



Apparatus Completely Assembled
Figure 2

diameter. It is supported by two self-aligning ball bearings which are 28 3/8 inches between centers. The bearings are supported in pedestals R.

The rotor S mounted at the center of the shaft is made up of suitable end pieces and a variable number of annular brass discs. The end pieces are 4 inches in diameter and have four ¼-inch holes equally spaced at a radius of 1½ inches from the center. Set screws in the collars of the end pieces secure the rotor to the shaft. The brass discs of the rotor are 4 inches in diameter with a section 2 inches in diameter removed and with four ¼-inch holes placed the same as in the end pieces. The discs and end pieces are held together by means of bolts through the ¼-inch holes. Thus the several discs form a hollow rotor. This reduces the restraint on the bending of the shaft. In order to make the rotor unbalanced only three bolts are used to hold it together.

A 50-ohm rheostat is used in the armature circuit to control the speed of the motor.

A "Strobotac", manufactured by the General Radio Company, is used for measuring speeds of the rotor. A hand tachometer was used on preliminary runs. It was discarded when it was found that the speed of the rotor changed 20 or more r.p.m. when the tachometer was applied to the end of the shaft.

A dial guage is used to measure deflections of the base. It is placed at the free end of the lower cantilever spring D and held in position by a stand which is made fast to the floor.

When it is desired to secure the mounting rigidly a section of 4-inch channel beam bent at right angles is bolted to the bottom of the vertical piece J and to a fitting in the floor. This gives a practically non-flexible mounting. No motion of the springs is discernible. There is, however, very slight flexibility in the base M.

LABORATORY PROCEDURE

The elastic constant of the mounting was varied by changing the length of the flat bars that serve as springs. The mass of the rotor was varied by changing the number of brass discs used.

For each change of either spring constant of the mounting or of mass of the rotor the following procedure was carried out. At various speeds from 100 to 1500 r.p.m. the amplitudes of vibration of the mounting and of the rotor were measured and recorded. Each of these runs was made with both increasing and decreasing values of speed. The intervals of speed at which readings were made depended on the proximity of the critical speed. At the critical speeds several checks of the speed and amplitude were made to insure greater accuracy.

Then with the base made rigid the speeds and amplitudes of vibration were measured and recorded with both increasing and decreasing values of speed.

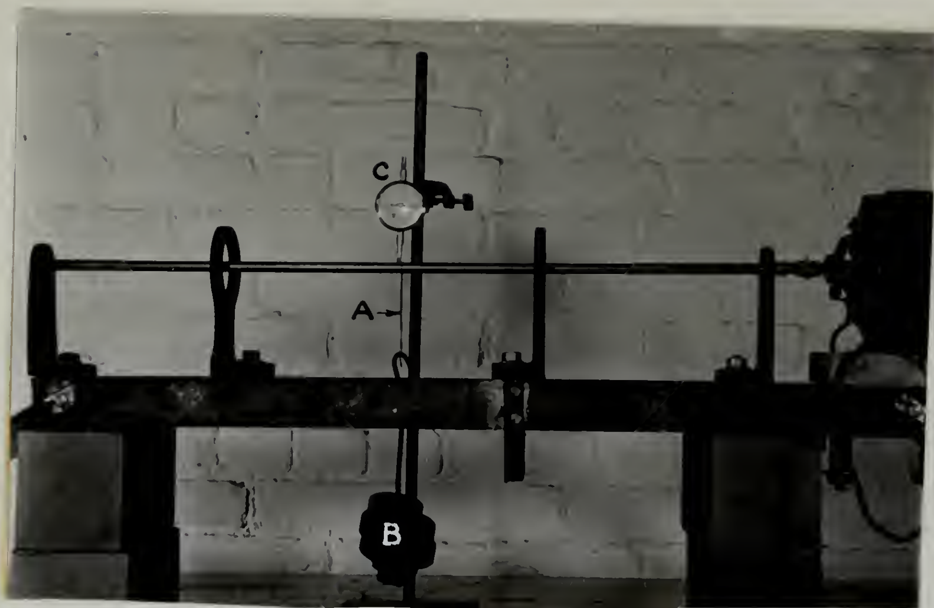
A cathetometer was used to measure the amplitude of the rotor. The instrument used had a least count of .0001 foot. Before starting a run a zero position was fixed by sighting the cathetometer at the topmost point on the rotor and then slowly rotating the rotor to check its position. Then at any desired speed the highest point of the rotor during vibration was sighted with the cathetometer. The difference between the two readings was recorded as the amplitude of vibration.

In obtaining the deflections of the base a dial guage with a least count of .001 inch was placed at the outer end of the lower spring. Thus at any speed the total movement of the base was read on the indicator.

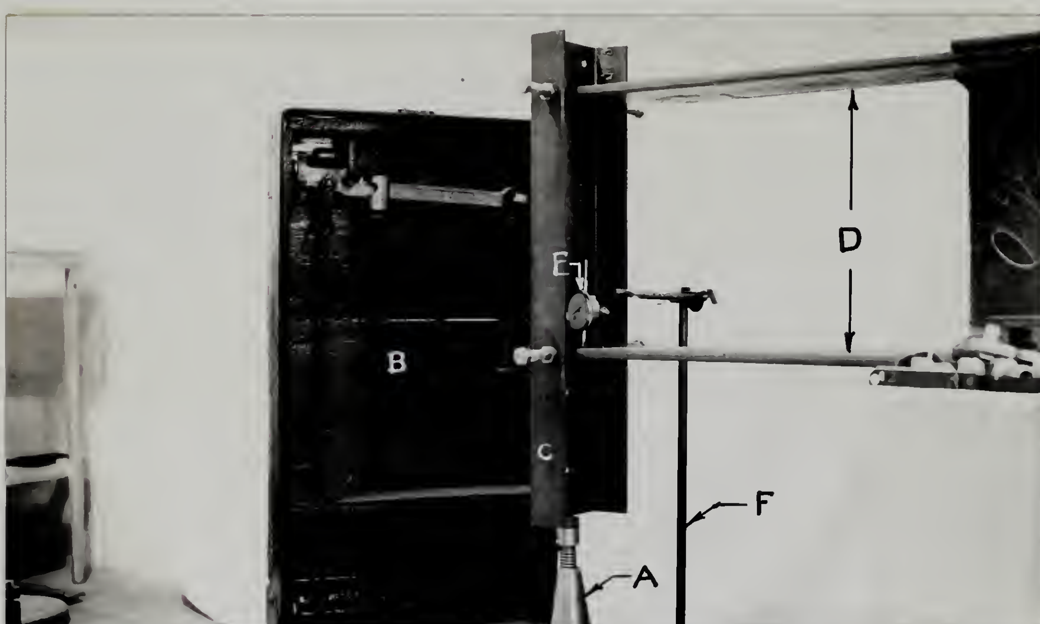
Speeds of revolution of the rotor were measured with a "Strobotac". This instrument was calibrated and kept regulated to read correctly.

The method used to determine the elastic constant of the shaft will be explained with the aid of figure 3. A loop of fine wire A was hung from the middle of the shaft. Various known weights were attached to the wire at B. Then the dial guage C placed at the middle of the shaft indicated the deflections caused by the weights. From the values obtained a curve of deflection versus applied weight was plotted. Then from this plot the elastic constant of the shaft was readily computed. Points on the curve were checked by rotating the shaft to different positions for each change of the weights. (See curve sheet 3.)

The elastic constant of the mounting was determined by a somewhat similar method. Figure 4 shows the apparatus used. A screw jack A was placed on the platform of the scales B. The top of the jack was made to exert force on the bottom of the vertical piece C which is attached to the cantilever springs D. Then as force was increased by extending the jack it was measured by the scales. The deflection of the springs was indicated by the dial guage E. This guage was held on stand F which was secured to the floor. Values obtained by this method were checked by deflecting the springs in the opposite direction with known weights placed on top of vertical piece C. Then a curve was plotted with deflection versus applied force



Apparatus For Determining The Elastic Constant Of The Shaft
Figure 3



Apparatus For Determining The Elastic Constant Of The Mounting
Figure 4

as coordinates. From this curve the elastic constant of the mounting was computed. This procedure was repeated for each length of cantilever springs used. (See curve sheet 4).

DERIVATION OF EQUATIONS

In making the derivation of the theoretical equations for the natural frequencies of the system under consideration the following conditions were considered to hold:

(1) The shaft and the cantilever springs are of uniform cross-section and homogeneous material.

(2) The shaft and the cantilever springs obey Hooke's law throughout the range over which they are deflected.

(3) In computing frequencies the effect of the distributed mass of the shaft may be accounted for by considering 17/35 of its amount to be concentrated at its middle with the rotor. This assumes a static deflection curve. (Reference 2, page 85.)

(4) In computing frequencies the effect of the distributed mass of the cantilever springs may be accounted for by considering 33/140 of their amount to be concentrated at their free end. This assumes a static deflection curve. (Reference 2, page 86.)

(5) Steady state motion prevails for all considerations taken up in this paper.

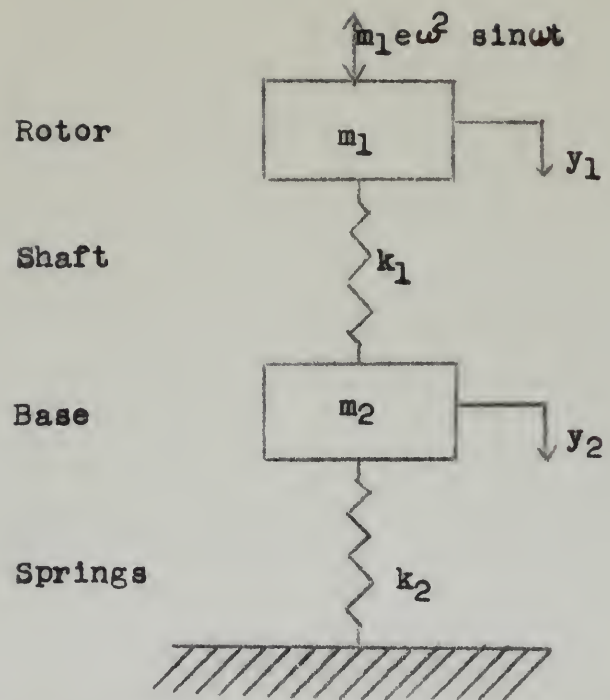
(6) Damping is negligible

Considering the system to be compound with two degrees of freedom it may be represented diagrammatically as in figure 5. The following symbols, subscripts, and units are used:

m_1 - mass of rotor including proper portion of shaft--lbs. sec.^{2}/in.

k_1 - elastic constant of shaft--lbs./in.

y_1 - displacement of the rotor--inches



Diagrammatic Representation of the System
Used in Making the Mathematical Analysis
Figure 5

a_1 - amplitude of vibration of rotor--inches

m_2 - mass of base including proper portion of cantilever springs--lbs. sec.^{2}/in.

k_2 - elastic constant of the mounting--lbs./in.

y_2 - displacement of base--inches

a_2 - amplitude of vibration of base--inches

e - distance between geometrical center of rotor and its center of gravity--inches

ω - frequency of the exciting force--radians/sec.

$m_1 e\omega^2 \sin t$ - exciting force due to unbalanced rotor

Then considering the solution for the amplitudes of vibration for the steady state, the following force equations may be written for the two masses:

$$-k_1(y_1 - y_2) \quad m_1 e\omega^2 \sin\omega t = m_1 \ddot{y}_1 \tag{1}$$

$$k_1(y_1 - y_2) - k_2 y_2 = m_2 \ddot{y}_2 \tag{2}$$

Assume as solutions:

$$y_1 = a_1 \sin\omega t \qquad\qquad y_2 = a_2 \sin\omega t$$

Then

$$\ddot{y}_1 = -a_1\omega^2 \sin\omega t \qquad\qquad \ddot{y}_2 = -a_2\omega^2 \sin\omega t$$

Substituting these values in equations 1 and 2,

$$-k_1 a_1 \sin\omega t + k_1 a_2 \sin\omega t + m_1 e\omega^2 \sin t = -m_1 a_1 \omega^2 \sin t \tag{3}$$

$$k_1 a_1 \sin\omega t - k_1 a_2 \sin\omega t - k_2 a_2 \sin\omega t = -m_2 a_2 \omega^2 \sin\omega t \tag{4}$$

Let $\omega t = \pi/2$. Then $\sin\omega t = 1$. Substituting in equations 3 and 4,

$$-k_1 a_1 + k_1 a_2 + m_1 e\omega^2 = - m_1 a_1 \omega^2 \tag{5}$$

$$k_1 a_1 - k_1 a_2 - k_2 a_2 = - m_2 a_2 \omega^2 \tag{6}$$

Transposing,

$$a_1(k_1 - m_1\omega^2) - a_2k_1 = m_1e\omega^2 \tag{5}$$

$$-a_1k_1 + a_2(k_1 - k_2 - m_2\omega^2) = 0 \tag{6}$$

Solving for a_1 by means of a determinant,

$$a_1 = \frac{\begin{vmatrix} m_1e\omega^2 & -k_1 \\ 0 & (k_1 - k_2 - m_2\omega^2) \end{vmatrix}}{\begin{vmatrix} (k_1 - m_1\omega^2) & -k_1 \\ -k_1 & (k_1 - k_2 - m_2\omega^2) \end{vmatrix}}$$

$$a_1 = \frac{m_1e\omega^2\,(k_1 - k_2 - m_2\omega^2)}{(k_1 - m_1\omega^2)\,(k_1 - k_2 - m_2\omega^2) - k_1^2} \tag{7}$$

If the denominator of the last expression is equal to zero the value of a_1 becomes infinite. This corresponds to a resonant speed.

Thus, $(k_1 - m_1\omega^2)(k_1 - k_2 - m_2\omega^2) - k_1^2 - 0$

Expanding and dividing through by m_1m_2,

$$\omega^4 - \left(\frac{k_1}{m_2} + \frac{k_2}{m_2} + \frac{k_1}{m_1}\right)\omega^2 + \frac{k_1k_2}{m_1m_2} = 0$$

Solving by the quadratic equation,

$$\omega^2 = \frac{1}{2}\left[\left(\frac{k_1}{m_2} + \frac{k_2}{m_2} + \frac{k_1}{m_1}\right) \pm \sqrt{\left(\frac{k_1}{m_2} + \frac{k_2}{m_2} + \frac{k_1}{m_1}\right)^2 - \frac{4k_1k_2}{m_1m_2}}\right] \tag{8}$$

In order to obtain the dimensionless expression for ω/ω_0 expand, take the square root, and divide the right hand side

of equation 8 by $\sqrt{k/m}$. Where $\omega_o = \sqrt{k/m}$ which is the critical speed of the rotor when the mounting is rigid.

$$\frac{\omega}{\omega_o} = \sqrt{\frac{1}{2}\left[\frac{m_1}{m_2} + \frac{k_2 m_1}{k_1 m_2} + 1 \pm \sqrt{\frac{m_1^2}{m_2^2} + \frac{2k_2 m_1^2}{k_1 m_2^2} + \frac{2m_1}{m_2} - \frac{2k_2 m_1}{k_1 m_2} + \frac{k_2^2 m_1^2}{k_1^2 m_2^2} + 1}\right]}$$

Then substitute $M = m_1/m_2$ and $K = k_2/k_1$.

$$\frac{\omega}{\omega_o} = \sqrt{\tfrac{1}{2}\left[(M + 1 + KM) \pm \sqrt{(M + 1)^2 + K(2M^2 - 2M) + K^2M^2}\right]} \qquad (9)$$

DISCUSSION

The agreement of the experimental values of resonant frequency with the computed values is very good as shown by curve sheets 1 and 2. The percentage difference of the experimentally found points from the computed curves varies from .5% to 4.5% with an average value of slightly less than 2%.

The near agreement of the points and the curves is considered as justification of the assumption that a rotor on an elastic shaft supported on an elastic mounting may be considered, in the ideal case, to be an elastically coupled two body system with two degrees of freedom in so far as vertical motion is concerned.

There are several reasons for the differences between the experimental and theoretical values of resonant frequencies. Personal and instrumental errors are always likely. It was, in some cases, difficult to measure the exact critical speed. This happened when the speed at which the amplitude of vibration of the rotor was unstable. In such instances the rotor would reach its greatest amplitude only momentarily and then go through the resonant condition and settle down to smooth running at a much higher speed. Another possible reason for the differences between the experimental and theoretical values is that friction and damping were present in various parts of the apparatus. The bearings were self-aligning but there was some friction in them. Also a small amount of pivot friction existed in the mounting.

There is a slight error (about 1%) produced in the formulas for critical speeds when portions of the shaft and of the cantilever springs are considered concentrated at points in order to correct for the distributed mass of these parts of the experimental apparatus. (See reference 2, page86.)

The rotor tends to stiffen the shaft thereby reducing its effective length. Also the slight bearing restraint has the same effect. The flexibility of the bearihg pedestals and of the base holding them tends to increase the effective length of the shaft. These two effects have apparently, at least partially, offset each other since by using the actual length of the shaft close agreement of experimental and theoretical results was obtained.

As noted in the derivation of equations damping was considered as having a negligible effect on the values of the critical speeds. However in the preliminary work the effect of damping was considered and investigated at some length. Den Hartog (reference 4, page 107) shows that relatively large values of the damping coefficient have little effect on the critical speeds. The magnitudes of the damping coefficients for the shaft and for the cantilever springs were obtained and found to be quite small. These values are tabulated on the data sheets.

In the analysis carried out in this paper vibrations in the vertical direction only are considered. However it should be noted that at various speeds the shaft was found to vibrate with the greatest amplitude in the horizontal plane. At still other

speeds the greatest amplitude of vibration was at an angle between the vertical and the horizontal. When constructing the laboratory apparatus it was made very rigid in the horizontal direction. Therefore the vibrations in the horizontal plane are very little affected by the flexibility of the mounting. Thus the former case above is the same as the critical speed if the mounting were rigid. The latter case represents a combination of the motion of the former case and the vertical motion existing when the base is free to vibrate.(See reference 2, page 212.)

Curve sheets 5 to 8 inclusive show that with the mounting rigid there is in each case a point in the neighboorhood of 450 r.p.m. where the amplitude of the rotor increases. The actual increase is slight, varying from 1/100 to 1/30 of an inch, and hence would be of little practical importance. A possible explanation for this increase in amplitude is found in reference 3, article 19b in which the author, Stodola, discusses the effects of the weight of the rotor, variable elasticity of the shaft, and variable driving impulse. Any or all of these effects are applicable to the apparatus used for test. Moreover in the same article Stodola mentions a situation practically the same as that under consideration here for which he could find no reason for the increase in the amplitude of vibration.

SAMPLE CALCULATIONS

First, to obtain values for the elastic constant of the cantilever springs, the following values were read directly from curve sheet 4:

f_2 (force) = 180 lbs. d_2 (deflection) = .198 inch

f_1 (force) = <u>10</u> lbs. d_1 (deflection) = <u>.007</u> inch

difference = 170 lbs.   difference = .191 inch

Therefore $k_2 = \frac{170}{.191} = 890$ lbs. per inch

In a similar manner the elastic constant for other lengths of the cantilever springs and for the shaft were computed.

Points for the theoretical curves of curve sheets 1 and 2 were obtained by substituting the experimental values of K and a series of arbitrary values of M into equation 9 which was derived in the " Derivation Of Equations".

Thus $k_1 = 106.6$ $k_2 = 890$ $K = \frac{890}{106.6} = 8.39$

Substituting this value of K and M - .1 into equation 9,

$$\omega/\omega_0 = \sqrt{\tfrac{1}{2}(1.100 + .839) \pm \sqrt{1.210 + .872 + 1.678)}}$$

$= 1.135$ or $.308$ (plotted on curve sheet 1)

The experimental data gave the following values from which an experimental point for curve sheet 1 is obtained.

$m_1 = .0192$ $m_2 = .205$ $M = \frac{.0192}{.205} = .0934$

$\omega = 820$ or 565 (from curve sheet 5)

$\omega_0 = 740$ (from curve sheet 5)

$\frac{\omega}{\omega} = 1.11$ or $.765$ (plotted on curve sheet 1)

The dimensionless coefficient of friction, γ, is given by the following expression:

$$\gamma = \frac{\log_e h_1/h_2}{\pi} = \frac{\log_e h_2/h_3}{\pi}$$

$$\gamma = \frac{2n}{\omega} = \frac{b}{m\omega}$$

b = coefficient of viscous damping force

h_1, h_2, h_3 = successive amplitudes of free vibration as shown on figure 6

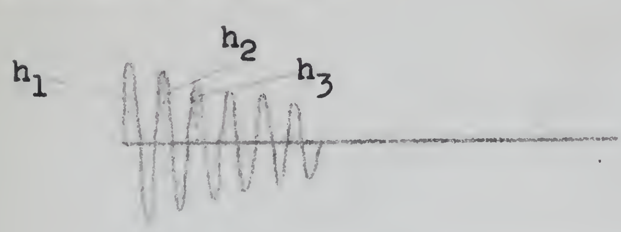
Thus the value of γ for the rotor when $m_1 = .0116$ is computed below.

$h_1 = .48$ inch $\qquad h_2 = .421$ inch

$h_1/h_2 = 1.142 \qquad \log_e 1.142 = .1327$

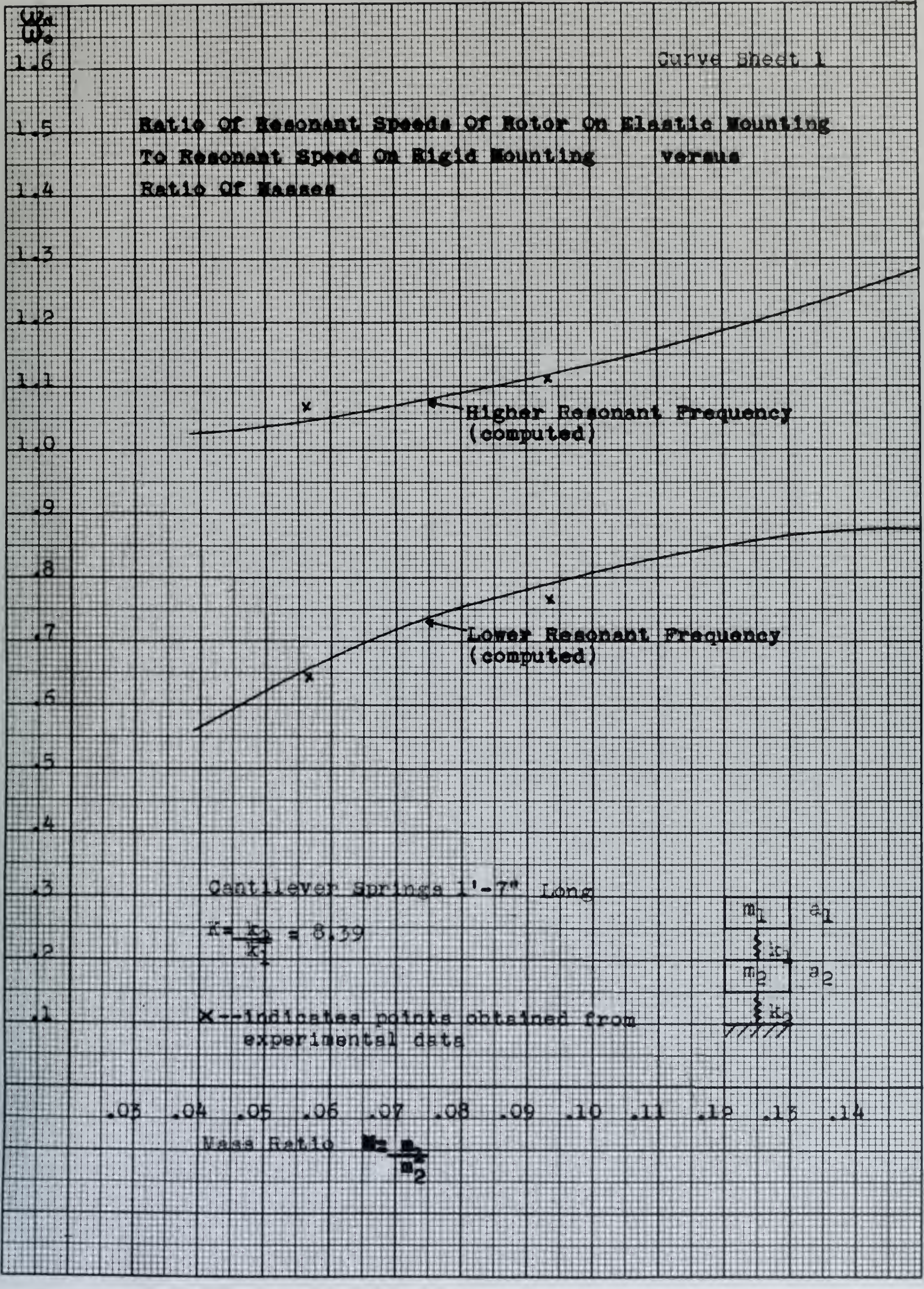
$\gamma = .1327/3.1416 = .0422$

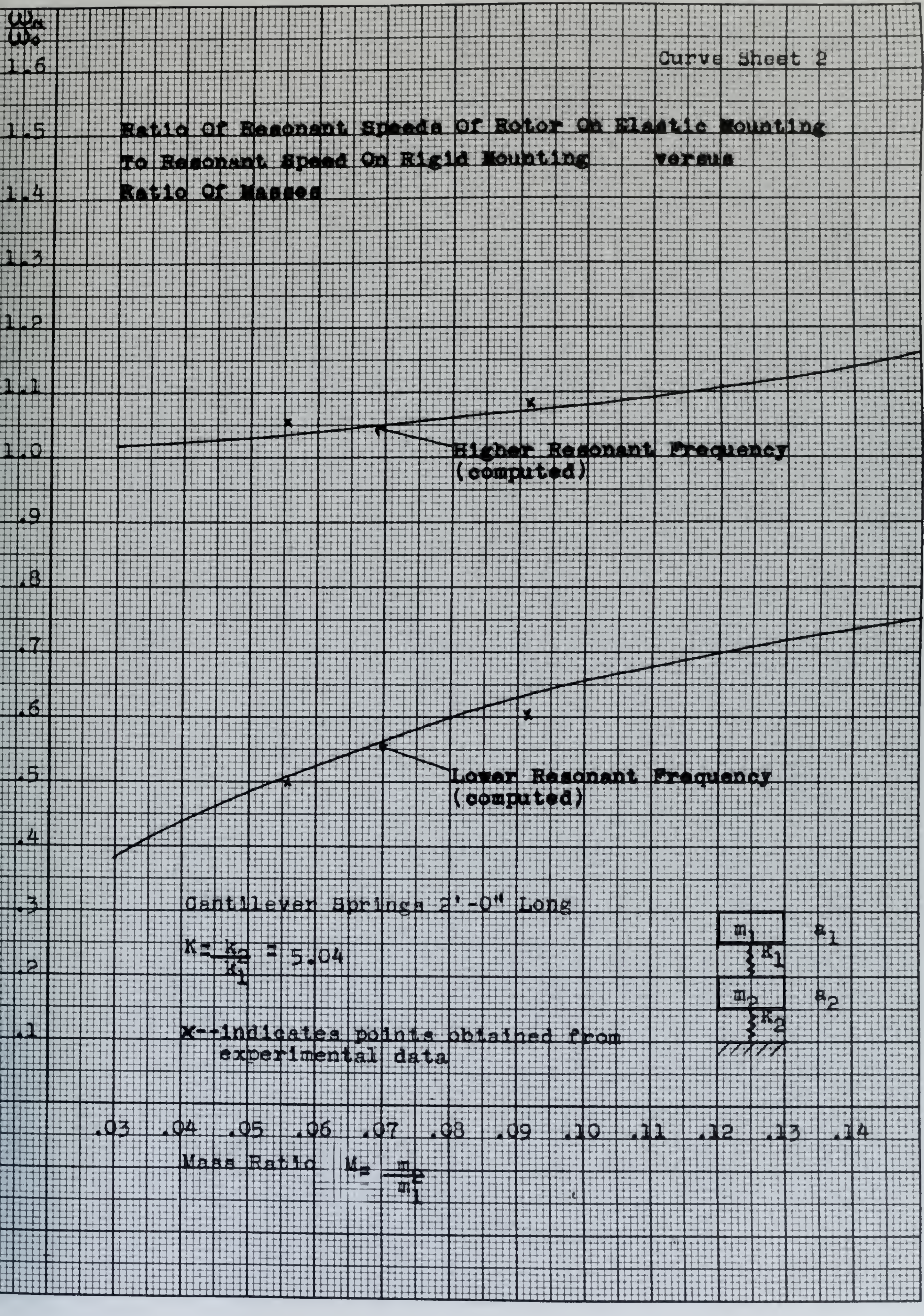
Values of γ for the rotor with a different value of m_1 and for the cantilever springs at the various lengths were similarly computed.



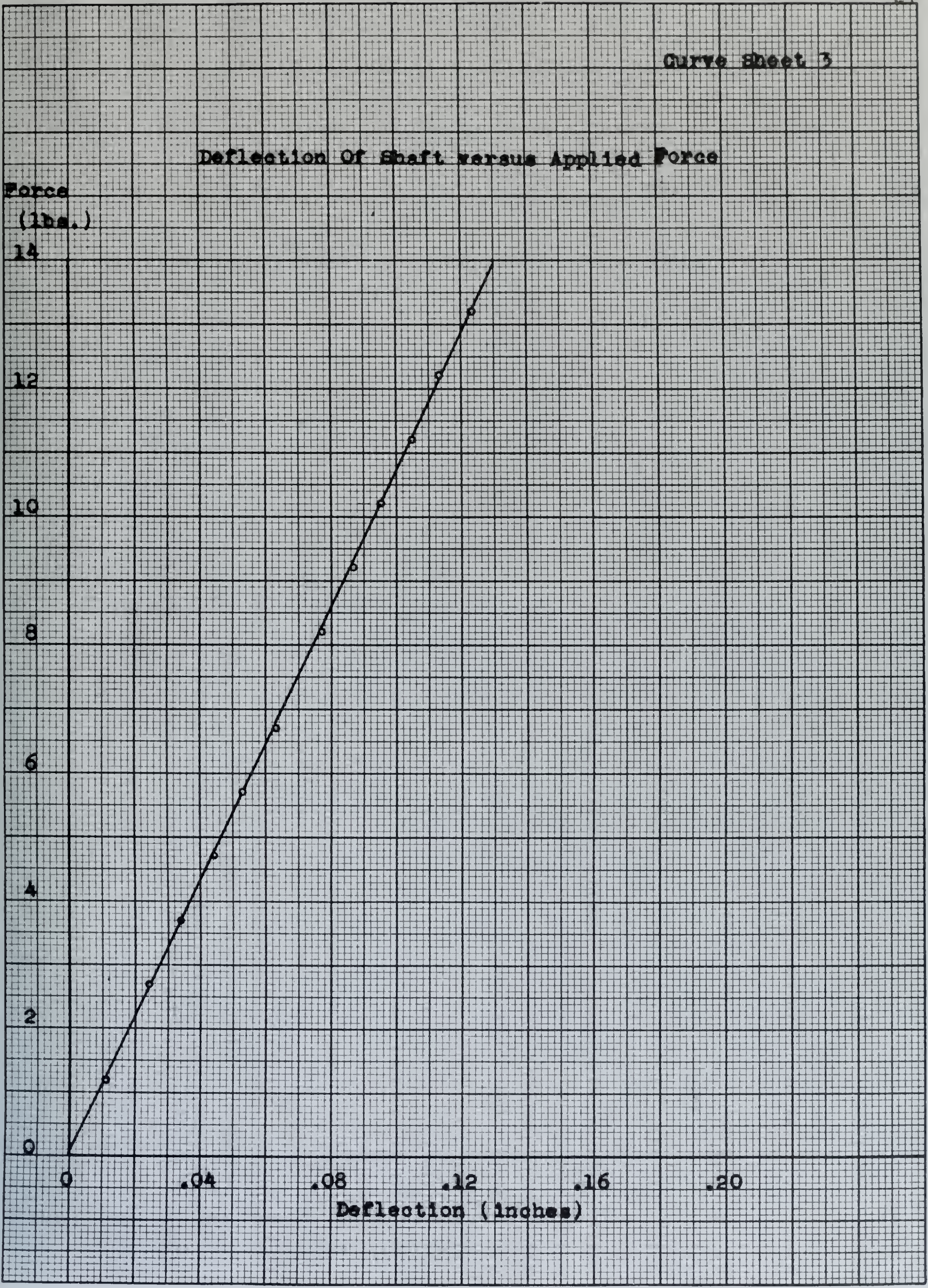
Record of Free Vibration Made by Vibrograph

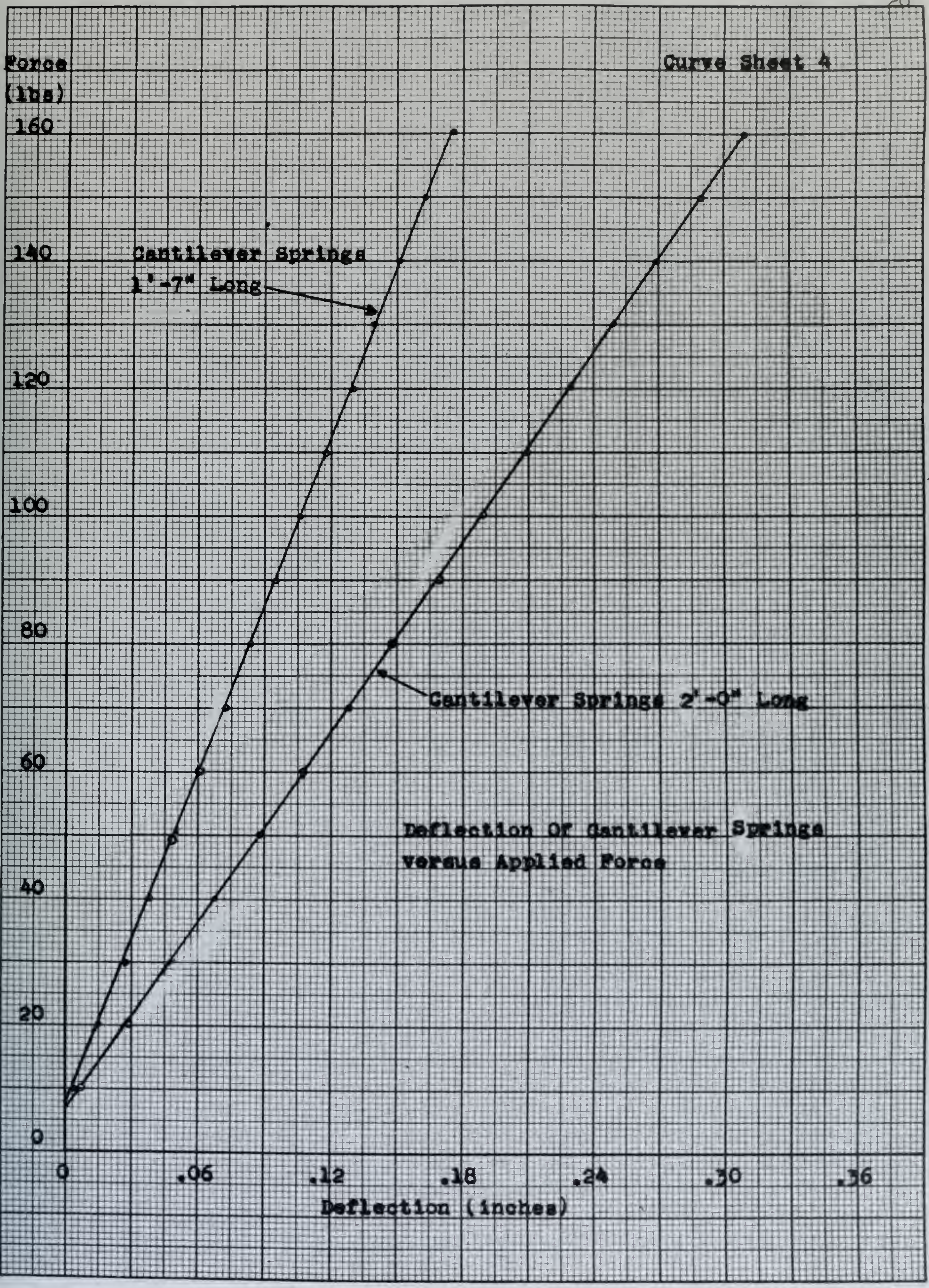
Figure 6


Curve Sheet 1
Ratio Of Resonant Speeds Of Motor On Elastic Mounting
To Resonant Speed On Rigid Mounting versus
Ratio Of Masses
$\frac{\omega_n}{\omega_0}$
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
Higher Resonant Frequency
(computed)
Lower Resonant Frequency
(computed)
Cantilever Springs 1'-7" Long
$K = \frac{k_2}{k_1} = 8.39$
X--indicates points obtained from
experimental data
m_1
a_1
k_1
m_2
a_2
k_2
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
.14
Mass Ratio $M = \frac{m_1}{m_2}$

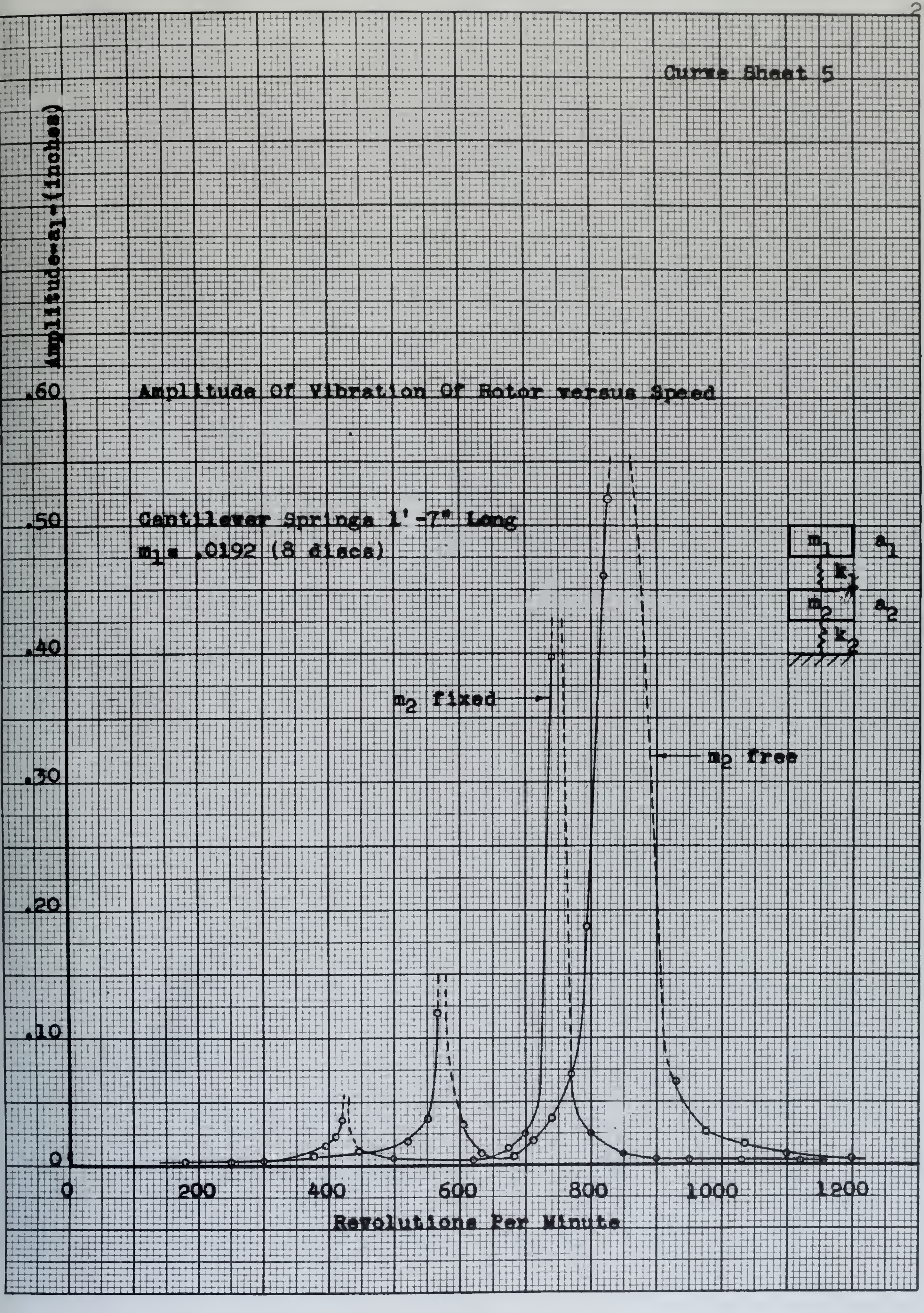


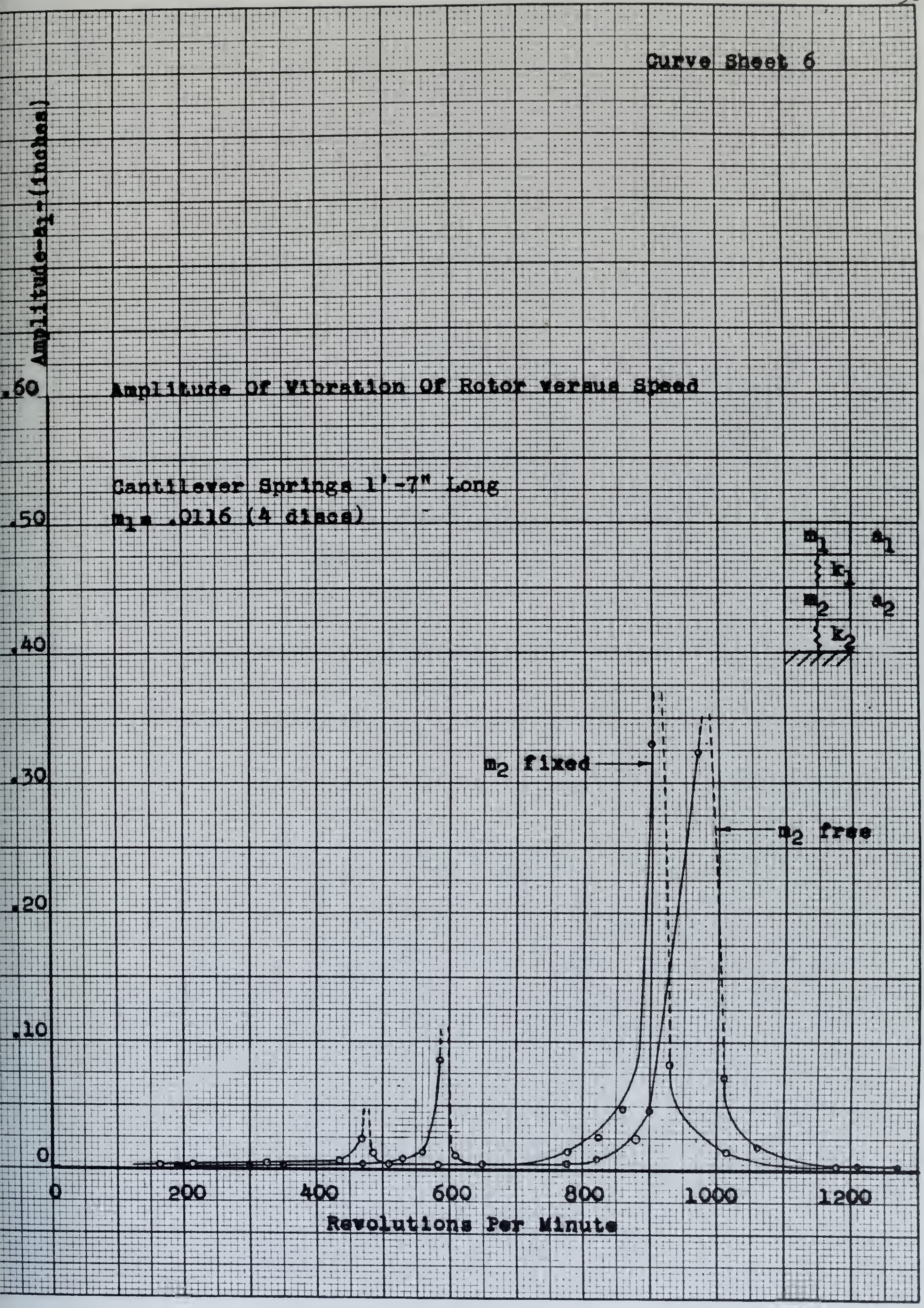
Curve Sheet 2
Ratio Of Resonant Speeds Of Rotor On Elastic Mounting
To Resonant Speed On Rigid Mounting versus
Ratio Of Masses
$\frac{\omega_n}{\omega_o}$
1.6
1.5
1.4
1.3
1.2
1.1
1.0
.9
.8
.7
.6
.5
.4
.3
.2
.1
Higher Resonant Frequency
(computed)
Lower Resonant Frequency
(computed)
Cantilever Springs 2'-0" Long
$K = \frac{k_2}{k_1} = 5.04$
X--indicates points obtained from
experimental data
m_1
k_1
m_2
k_2
a_1
a_2
.03
.04
.05
.06
.07
.08
.09
.10
.11
.12
.13
.14
Mass Ratio $M = \frac{m_2}{m_1}$

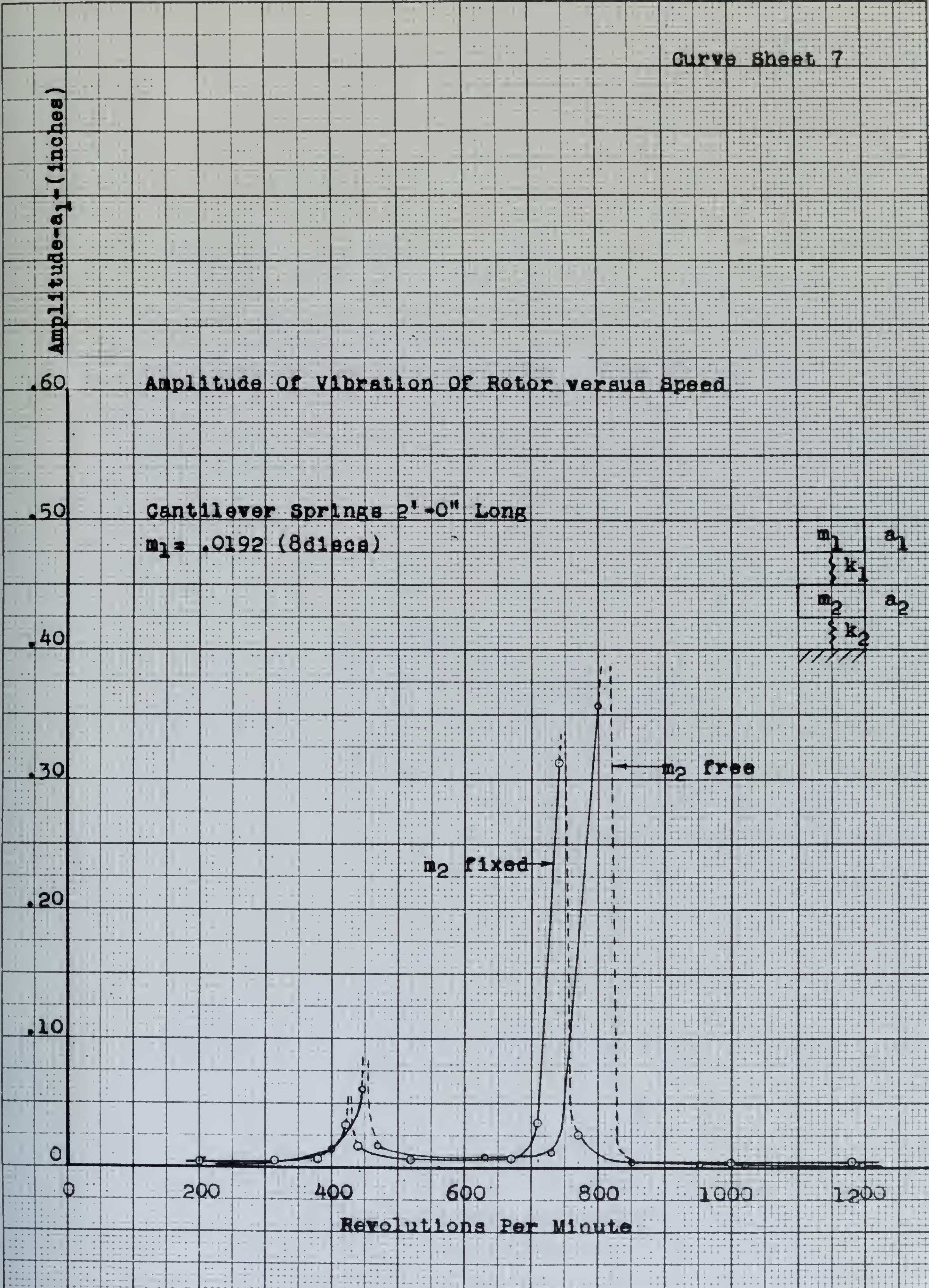

Curve Sheet 3
Deflection Of Shaft versus Applied Force
Force (lbs.)
14
12
10
8
6
4
2
0
0
.04
.08
.12
.16
.20
Deflection (inches)



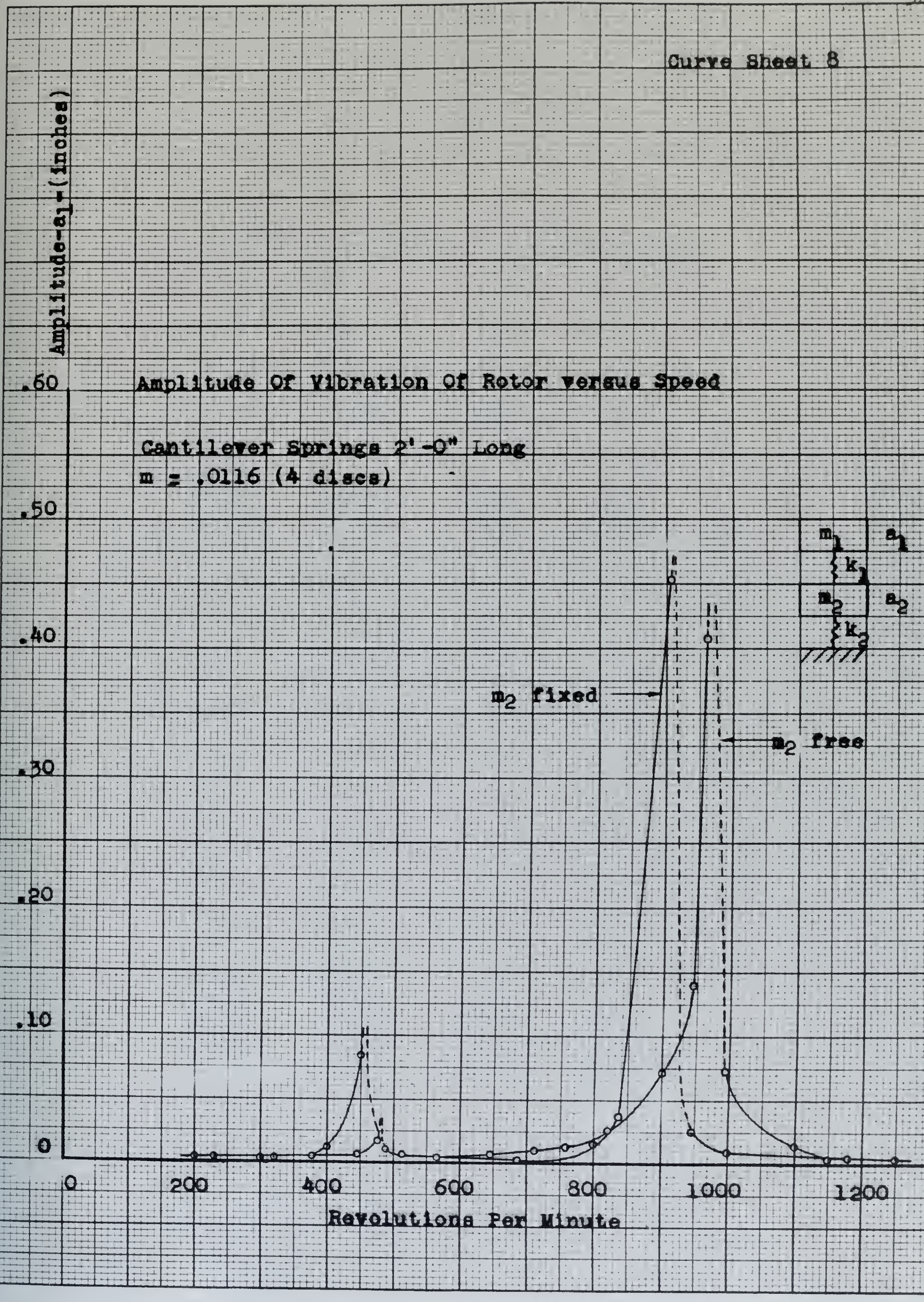
Force
(lbs)
Curve Sheet 4
160
140
120
100
80
60
40
20
0
Cantilever Springs
1'-7" Long
Cantilever Springs 2'-0" Long
Deflection Of Cantilever Springs
versus Applied Force
0
.06
.12
.18
.24
.30
.36
Deflection (inches)


Curve Sheet 5
Amplitude-a1-(inches)
Amplitude Of Vibration Of Rotor versus Speed
Cantilever Springs 1'-7" Long
m1= .0192 (8 discs)
m1
a1
k1
m2
a2
k2
m2 fixed
m2 free
.60
.50
.40
.30
.20
.10
0
0
200
400
600
800
1000
1200
Revolutions Per Minute


Curve Sheet 6
Amplitude Of Vibration Of Rotor versus Speed
Cantilever Springs 1'-7" Long
m1= .0116 (4 discs)
Amplitude-a1-(inches)
.60
.50
.40
.30
.20
.10
0
m1
a1
k1
m2
a2
k2
m2 fixed
m2 free
0
200
400
600
800
1000
1200
Revolutions Per Minute



Curve Sheet 7
Amplitude-a_1-(inches)
Amplitude Of Vibration Of Rotor versus Speed
Cantilever Springs 2'-0" Long
m_1= .0192 (8discs)
m_1
a_1
k_1
m_2
a_2
k_2
m_2 free
m_2 fixed
.60
.50
.40
.30
.20
.10
0
0
200
400
600
800
1000
1200
Revolutions Per Minute



Curve Sheet 8
Amplitude Of Vibration Of Rotor versus Speed
Cantilever Springs 2'-0" Long
m = .0116 (4 discs)
Amplitude-a_1-(inches)
.60
.50
.40
.30
.20
.10
0
m_1
a_1
k_1
m_2
a_2
k_2
m_2 fixed
m_2 free
0
200
400
600
800
1000
1200
Revolutions Per Minute

DATA

Values of r.p.m. and corresponding amplitude of vibration of the rotor. Amplitudes are recorded in fractions of an inch.

Cantilever Springs 2'-0" Long

m_1 = .0192

Fixed Base		Free Base	
r.p.m.	amplitude	r.p.m.	aamplitude
200	.0036	200	.0036
315	.0048	250	.0036
380	.0060	370	.0060
410	.0190	400	.0140
420	.0330	445	.0600
440	.0150	465	.0140
520	.0060	520	.0096
600	.0060	630	.0072
670	.0060	730	.0120
710	.0400	800	.3550
740	.3130	850	.0048
770	.0036	950	.0024
1000	.0036	1000	.0024
1100	.0036	1120	.0024
1180	.0036	1190	.0024
1250	.0036	1250	.0024

Cantilever Springs 2'-0" Long

m_1 = .0116

Fixed Base		Free Base	
r.p.m.	amplitude	r.p.m.	amplitude
200	.0024	230	.0024
320	.0024	300	.0048
400	.0060	400	.0120
445	.0060	450	.0840
475	.0180	485	.0072
485	.0120	510	.0072
530	.0048	560	.0072
600	.0012	645	.0072
675	.0012	710	.0120
750	.0012	755	.0132
810	.0150	800	.0156
835	.0360	8200	.0300
905	.4530	900	.0720
945	.0240	945	.1400
1000	.0096	960	.4070
1150	.0036	995	.0720
1200	.0036	1040	.0480
		1100	.0120
		1170	.0024

Cantilever Springs 1'-7" Long

m_1 = .0192

Fixed Base		Free Base	
r.p.m.	amplitude	r.p.m.	amplitude
180	.0024	200	.0036
250	.0036	300	.0048
300	.0036	375	.0072
395	.0152	455	.0096
410	.0240	520	.0192
420	.0360	550	.0360
445	.0072	565	.1200
500	.0060	605	.0320
620	.0048	640	.0054
675	.0120	680	.0072
700	.0240	710	.0240
740	.3950	740	.0360
770	.0720	795	.1860
800	.0240	820	.4620
850	.0096	930	.0660
900	.0048	975	.0240
950	.0048	1035	.0180
990	.0048	1100	.0072
1030	.0048	1150	.0048
1120	.0036	1200	.0048

Cantilever Springs 1'-7"

m_1 = .0116

Fixed Base		Free Base	
r.p.m.	amplitude	r.p.m.	amplitude
165	.0024	190	.0024
215	.0036	300	.0024
325	.0036	350	.0036
435	.0054	470	.0036
470	.0240	530	.0052
480	.0120	560	.0048
510	.0048	585	.0840
585	.0048	600	.0048
650	.0048	690	.0048
700	.0084	775	.0048
740	.0120	820	.0084
780	.0144	880	.0144
835	.0240	900	.0480
860	.0480	970	.3230
900	.3300	1010	.0720
930	.0820	1060	.0180
970	.0240	1100	.0120
1015	.0120	1160	.0096
1080	.0054	1210	.0048
1180	.0036		

DATA

Values of γ found experimentally.

(γ = dimensionless coefficient of damping)

Cantilever springs	Value of γ
1 foot 7 inches long	.0915
2 feet 0 inches long	.0633

Rotor

m_1 = .0116	.0422
m_1 = .0192	.0294

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Thesis 6318
M2 MacKay
The effect of flexible mountings upon the resonant speeds of machines having unbalanced motors.